Logic in Pragmatics*

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This paper argues against the total elimination of logical introduction rules from the pragmatic inference system. To maintain consistency of the inference system as a whole, which is meant to support one’s truth-based judgment over propositions, the inference system should have access to both introduction and elimination rules. I show that the inclusion of introduction rules into the pragmatic inference system neither overgenerates propositions expressed nor causes non-terminating inference steps.

1 FREE ENRICHMENT AND ALLEGED OVERGENERATION

According to Relevance Theory (RT for short, Sperber & Wilson 1986/95), the pragmatic process of recovering propositions expressed by (or the truth conditional content of) utterances involves more than reference assignment and disambiguation.

(1) (a) Every presenter [in the pragmatics session of CamLING07] was impressive.
     (b) John took out a key and opened the door [with the key]. Cf. Hall (2006)

Given the linguistically provided information outside the square brackets in (1), the hearer can pragmatically enrich to propositions that may contain extra material such as the content given in the square brackets (in the relevant contexts).

Stanley (2002) claims that this free enrichment overgenerates. Suppose (2) is uttered with the contextual premises in (3) being highly accessible. Then, according to Stanley, RT wrongly predicts that (2) can be enriched to (4), assuming that John will not live long (=R) is a relevant conclusion to draw.

(2) John smokes. (=P)
(3) a. John drinks. (=Q)
    b. If John smokes and drinks, he will not live long. (= (P&Q)→R)
(4) John smokes [and drinks]. (=P&Q)

Addressing this criticism, Hall (2006: 95-96) follows Sperber & Wilson (1986/1995) and postulates Conjunctive Modus (Ponendo) Ponens (CMPP) as in (5). With CMPP, the hearer can derive the relevant conclusion, John will not live long (=R), without applying &I.

(5) Conjunctive Modus Ponens:
    1. (P&Q)→R Premise 1
    2. P Premise 2
    3. Q→R 1, 2, CMPP
    4. Q Premise 3
    5. R 1, 2, 4, MPP

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Hall suggests a weaker claim such that because of CMPP, the hearer does not have to use &I in order to derive the relevant conclusion *John will not live long*. Thus, the hearer can derive this conclusion as in (5), without deriving the undesirable (4) as the proposition expressed by (2).

However, with this weaker claim, to prohibit the derivation of (4) as the truth condition of (2) in any instance of interpreting (2) in context, one would need some additional explanation why the hearer *always* uses the inference steps as in (5) rather than the application of &I followed by MPP, when free enrichment is involved.

In this paper, I argue that introduction rules can be used in pragmatic inferences in general. Thus, after showing that it is problematic to eliminate the &Introduction rule from the pragmatic inference system in section 2, I provide an explanation in section 3 about why &I is not used in enrichment, though the pragmatic inference system itself is equipped with this rule. I also argue that CMPP is only a convenient shorthand for a particular combination of inference steps, rather than an actual inference rule defined over logical connectives.

The stronger claim made by Sperber and Wilson is that spontaneous inferences do not use (logical) introduction rules at all (and thus, (5) is the only way of deriving the conclusion *R*, given (2)-(3)). The reason for postulating this stronger hypothesis is not only the alleged overgeneration of propositions expressed by way of free enrichment. Sperber & Wilson, among others, argue that spontaneous inference should not have access to introduction rules because, otherwise, the system would generate infinite or non-terminating inferences. In section 4, I briefly explain this infinity problem and then show that the problem is not caused by the use of introduction rules in the system, and thus eliminating &I or other introduction rules is not the right way of coping with this problem. Section 5 is an Appendix in which I present some proofs to support my arguments. Section 6 provides concluding remarks.

## 2 PROBLEMS OF ELIMINATING &I FROM THE PRAGMATIC INFERENCE SYSTEM

In this section, I discuss some of the problems of eliminating &I from the spontaneous inference system. First, if the truth-based judgment is at least part of one’s spontaneous inference, then the total inference system might become inconsistent without &I.

\[
\begin{align*}
(6) & \quad (a) \quad p, q, (p \& q) \rightarrow r \vdash r \\
& \quad (b) \quad p \& q, (p \& q) \rightarrow r \vdash r
\end{align*}
\]

Consider the two sequents in (6a) and (6b). The two separate propositions *p* and *q* on the one hand, and one complex proposition (*p* \& *q*) on the other, have the same interpretation in the antecedents of the two sequents. If the inference system cannot make use of &I, it cannot syntactically explain the same role that these formulas play in truth-based interpretations.\(^2\) Secondly, the reason why CMPP in (5) does not cause problems for the logical inference system as a whole is the logical equivalence relation in (7), whereas the proof of this equivalence requires &I, as well as &Elimination (&E).

\[
(7) \quad (p \& q) \rightarrow r \vdash r \quad p \rightarrow (q \rightarrow r)
\]

Though humans may not actually derive the right-hand side formula from the left-hand side one in (7) when they run an inference as in (5), inability to recognize the two as truth conditionally equivalent is a demerit of the inference system.

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\(^1\) As a ‘stronger claim,’ I include one that assumes that, though &I is available at the level of the underlying logical inference system, it is *never* used in any spontaneous inferences for some application reasons.

\(^2\) The inference system can still derive the entailment relation from (*p* \& *q*) to *p*, *q* (as premises) via &Elimination in the syntax, but that is not complete.
In this section, I showed that the truth conditional equivalence between certain propositions cannot be recognized without &I. Though rules that are used in pragmatic inference do not have to be complete with regard to the intended interpretations (i.e. truth-based interpretations in this case), assuming that such truth conditional judgments are also important in spontaneous inferences, it is not explanatory (as well as stipululatory without independent explanation) to ban the use of &I in the pragmatic inference system.

3 Alleged Over-generation by way of enrichment

In this section, I show that RT does not have to eliminate &I to prevent the alleged overgeneration via free enrichment.

In propositional logic, &I requires as premises two formulas that can be assigned truth values, as is informally shown in (8).

(8) (a). Syntax: \( p, q \vdash \&I \ p \& q \)
(b). Semantics: If \( ||p|| = \text{True} \) and \( ||q|| = \text{True} \), then it follows that \( ||p \& q|| = \text{True} \).

Because of this, if it is also assumed that the proposition expressed by an utterance is the first truth-evaluable meaning representation that can be derived from the language expression used, it follows that &I (or introduction rules for any truth functional connectives) cannot be used in the derivation of the proposition expressed. Consider (1)–(4) again. (3a, b) as contextual assumptions are fully propositional on their own. On the other hand, (2), which is uttered by the speaker, acquires a fully propositional status only after it is recognized as the proposition expressed by that utterance. Thus, one can conjoin (2) with (3a) via &I only after recognizing (2) on its own as the proposition expressed. It follows that (4) cannot be the proposition expressed by (2). Note that in this explanation, CMPP is not required as an actual logical inference rule. CMPP may still be used for describing an on-line inference step that arises as a result of routinization of certain logical inference steps in application. But that does not cause any inconsistency in the inferential system as a whole. With &I, the system can recognize the equivalence between the role of the two premises \( p \) and \( q \) separately, on the one hand, and the role of \( (p \& q) \) as one complex premise, on the other.

I stipulated that one can apply introduction rules for truth conditional connectives only after one enriches the meaning of the overtly used expression to the proposition expressed. The proposal would be problematic if one had to apply a truth based logical inference rule to a propositional representation that has not yet been accepted as the proposition expressed in other well-attested cases. Some might argue that ‘trivial propositions’ as in (9) are such cases.

(9) (a). John has a brain. (i.e. John is smart.)
(b). Meg is human. (i.e. Meg may make mistakes, etc.)

An argument against my proposal above would be that, in order to derive the propositions expressed (e.g. the ones in the parentheses in (9a) and (9b)), the hearer has to evaluate the literal meanings of (9a) and (9b) as trivially true propositions.

However, (9a) and (9b) do not necessarily provide a problem for my proposal. First, to recognize the literal meanings of (9a) and (9b) as trivially true, one does not have to apply proper logical inference rules. In other words, to recognize them as trivially true, one does not

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3 Hall (2006) postulates two kinds of pragmatic inferences, local inferences that are applicable to sub-propositional expressions, and global inferences that apply only to fully propositional expressions. Allott (p.c.) suggests that this division might inherently be present in Sperber & Wilson (1986/1995). One can regard my proposal in section 2 as an interpretation of this division between two kinds of inferences.
have to have the trivially true propositions interact with other contextual assumptions in terms of logical inference rules.4

Secondly, after accepting my assumptions, one can still enrich the meaning of component expressions by using the information provided by contextual assumptions. With model theoretic relations such as sub-set relations, one can mimic logical entailment relations without deriving a fully propositional representation. In this way, one can enrich the meanings of predicate expressions via set-containment relations, for example, without deriving a full proposition. On the other hand, I argue that proper logical introduction rules do require fully truth evaluable elements as premises (just as is the case in standard logical systems) and thus, they cannot be mimicked in terms of relations between sets.5

This section explained why introduction rules are not applicable in enrichment. The next section deals with the alleged ‘infinite inference’ problem.

4 ALLEGED INFINITY PROBLEM CAUSED BY INTRODUCTION RULES

This section briefly addresses the claim that if one’s pragmatic inference system were equipped with logical introduction rules, one would run infinite or non-terminating inferences. Because such non-terminating inferences are not attested in interpretation data, it must be the case that the pragmatic inference system does not have access to introduction rules. Arguing against such claims, I show that this problem is caused independently of the use of introduction rules and should be solved independently.

Johnson-Laird (1997: 391) claims that introduction rules, if they are used in spontaneous inferences, may lead to infinite inference steps, as schematized in (10).

(10) (a). \[ P, Q \vdash \&I P \& Q \vdash \&I P \& Q \vdash \&I \ldots \]
(b). \[ P \vdash \lor I P \lor Q \vdash \lor I P \lor Q \lor R \vdash \lor I \ldots \]

However, the alleged infinity in (10a) is because of the expansion of ‘P’ to ‘P, P,’ and ‘Q’ to ‘Q, Q.’ It is not because of \&I per se. Also, with regard to this structural expansion rule, note that one occurrence and more than one occurrence of the same formula have the same interpretation in truth-based inferences. Thus, the alleged infinity might be just a matter of an imperfect representation system, rather than some imperfection of the inference system. In fact, even at the level of represented deductions, logicians have tried to eliminate undecidability introduced by structural rule applications. Without going into the details, the idea is that one may apply a structural rule only when the consequence of that rule application is required by the next step of the inference. In (10b), \lor I presupposes weakening on the succedent side. But this weakening is only required to make the syntactic system complete with regard to the truth-based interpretations. In other words, though Q and R are introduced as disjuncts on the succedent side, Q and R on their own are not usable as a premise at a later stage of derivation. In that sense, P, P\lor Q and P\lor Q\lor R have the same interpretation in this inference and could well be recognized as the same in the truth-based inference.

Some might argue that recursive applications of \&Introduction followed by \&Elimination would produce infinite inference steps, but this infinity does not arise in standard proof representations without a ‘Cut’, such as Gentzen sequent presentation without a Cut. Some proofs are listed in the Appendix (see (16)–(19)).

(11) schematically shows a more sophisticated infinity argument.

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4 Such propositional interactions are not necessary, even when the literal meanings of (9a, b) are recognized as informative enough and are accepted as propositions expressed. Contextual premises, plus the linguistic meanings of the relevant expressions, will provide enough clues without such interactions.

5 I regard generalized conjunction as in Partee and Rooth (1983) only as a rule of PF-LF mapping and it does not influence the fully truth-functional status of \&, \lor, \rightarrow, etc. at the level of logical forms.
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(11) Non-frame problem.

(a) Antecedent Set ⊢ Δ Succedent Set
(b) \( P \vdash \{Q, &I \} P & Q \)
(c) cf. \( P, Q \vdash &I P & Q \)

(11a) represents a spontaneous on-line inference step. Though the logical inference rules are the same as in classical logic, (11a) distinguishes between two kinds of databases that are used as premises. The antecedent embodies the set of premise propositions that are active in the context, including the proposition expressed by the utterance. To draw a conclusion in the Succedent Set, one can also use premise propositions in the ‘dormant database’ set \( \Delta \), which contains all of one’s (propositional) knowledge.

With these assumptions, some might argue that the inference system would wrongly predict the existence of an infinite inference as in (12).

(12) \( P \vdash \{Q, R, S, \ldots \} &I P & Q \vdash \{R, S, \ldots \} &I P & Q & R \vdash \{S, \ldots \} &I P & Q & R & S \vdash \{\ldots \} \ldots \)

In (12), one may extract one proposition after another from the dormant database set and conjoin them with the proposition \( P \) in the active premise set. If it is assumed that the amount of knowledge that one has (or can access) is almost infinite, this model wrongly predicts that one may actually run an almost infinite inference.\(^6\)

However, note that this alleged infinity is not a matter of \&I per se. As I have already pointed out, in the antecedent set, \( P \) and \( Q \) as separate propositions on the one hand, and \( P & Q \) as a single complex proposition on the other, play the same role in the classical logic. Thus, the above infinity problem will arise independently of the use of \&I. What is problematic then is the introduction of \( Q, R, S \) into the active data-base, not the conjunction of those newly introduced propositions with a proposition that is already in the active database. Thus, what one needs is a systematic way of constraining the introduction of propositions from the dormant database to the active database.

This section has shown that use of introduction rules in the inference system is not the cause of the alleged non-terminating inferences, and that the elimination of introduction rules does not solve the problem.

5 APPENDIX: SOME GENTZEN SEQUENT PROOFS

In this section, I show that successive applications of \&I (or \&R in this section) and \&E (or \&L) do not lead to undecidability. I also show that \((p \& q) \implies r \) and \( p \implies (q \implies r) \) are inter-derivable. The proofs here are elementary, and are a simple application of the Gentzen sequent presentation of classical logic as in Girard (1987) or Takeuti (1987).

(13) Sequent to prove (e.g.) \( p, q, (p \& q) \implies r \vdash r \)

The Gentzen sequent proof representation places the sequent to prove at the bottom of the derivation. Then, one logical connective after another is eliminated upwards along the chain, as is shown in below examples. If the proof is successful, the sequents at the top of the proof are all identity axioms in the form of (14).

(14) Axiom: \( A \vdash A \)

\(^6\) Also, in a spontaneous inference, one does not typically access all of one’s available pieces of knowledge, even if the pieces of knowledge are relevant to the argument being made.
By convention, $p, q, r \ldots$ represent atomic propositional letters, $A, B, C$ represent any (propositional) formulas, and $X, Y, Z$ represent sets of such formulas. I omit the set notations both in the antecedent (i.e. the left-hand) side of each turnstile and the succedent (i.e. the right-hand) side. (15) shows the axioms for the connectives $\&$ and $\rightarrow$. I omit rules for other connectives. Cut in (15c) is an admissible rule\(^7\) which is not necessary for the proof system, but is useful for improving the efficiency of the proof.

(15) Logical rules:

(a) \[
\frac{A, B}{X} \quad \frac{A \land B}{X} \]

(b) \[
\frac{X \quad A, Y, B}{Z} \quad \frac{X, Y, A \rightarrow B}{Z} \]

(c) \[
\frac{X \quad A}{A} \quad \frac{A}{Z} \quad \text{Cut} \]

I have omitted some of the ‘contextual’ structural variables (i.e. $X, Y, \ldots$). Except for the Cut rule, the number of the connectives decreases by one along each consecutive step upwards. Because there are only a finite number of connectives in each sequent that is to be proven, each proof is decidable in a finite number of steps, unless Cut is used. Remember the successive use of $\&I$ ($= \&R$ here) and $\&E(= \&L)$, which may allegedly lead to an infinite inference. With Cut, this claim is substantiated, as in (16).

(16) Proof 1

\[
\frac{p \quad q}{\frac{p \land q}{r}} \quad \frac{p \land q}{(p \land q) \rightarrow r} \quad \text{Cut} \]

(17), in which $\Gamma$ and $\Delta$ represent the two sub-proofs of (16), represents the proof in (16) in brief. If the Cut rule is used, then this proof might not terminate in a finite step, given the sequent to prove, $p, q, (p \land q) \rightarrow r \vdash r$.\(^8\)

(17) Proof 1 (with abbreviation)

\[
\frac{\Gamma}{p, q, (p \land q) \rightarrow r \vdash r} \quad \text{Cut} \]

In the position of the sub-proof $\Gamma$ in proof 1, one could insert a larger sub-proof, e.g., the whole of the proof 2 in (18).

\(^7\) That is, any sequent that can be proved with Cut is provable without Cut.
Note that the left premise and the conclusion of Cut in (18) are both \( p, q \vdash p \land q \). Thus, this conclusion sequent can be used as a left premise of another Cut, whereas the whole of the right premise of the original Cut in (18) is repeated as the right premise of the additional Cut. Thus, there is no maximal limit to the size of the sub-proof in (18), leading to the infinity (or undecidability) problem.

However, as Girard (1987) and others showed, Cut is an admissible rule in Gentzen sequent presentation. Without Cut, Proof 1 is represented as Proof 3.

Other than Cut, all the rules in (14)–(15) reduce the number of connectives by one along each consecutive step upwards, and thus, all the proofs are decidable in finite steps. Consequently, successive use of \( \land L \) and \( \land R \) does not lead to an infinite inference.

Finally, (21) shows that the equivalence in (7), repeated here as (20), is provable only with \( \& I \) (or \( \land R \) here) as a rule of the logic. The proof in (21a) requires \( \land R \) in the top left sub-proof.

In this section, I showed that successive use of \( \& I(=\land R) \) and \( \& E(=\land L) \) does not lead to an infinite inference in Gentzen sequent presentation without Cut. I also showed that \( \& I \) is a necessary inference rule to support CMPP as an application rule in spontaneous inference. I did not show how one can prevent infinity which could be induced by the use of structural
rules (such as expansion and weakening) in the proof presentations, but for some rough ideas (in the context of Modal logic), see Hudelmaier (1996).

6 Conclusion

If a pragmatic inference system is to explain the truth-based inference (possibly among other kinds of inference), it is not desirable to eliminate logical introduction rules completely from the inference system, assuming that the inference system needs to be consistent as a whole. Use of introduction rules in the inference system as a whole does not lead to overgeneration via enrichment. Introduction rules can apply only with fully propositional elements as premises, and thus, such rules cannot be applied before the recovery of the proposition expressed. The alleged infinite inference steps are not caused by introduction rules per se, and the problem must be solved independently.

References


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